# *DENSE NEUTRINO OSCILLATIONS: BEYOND TWO FLAVOR*

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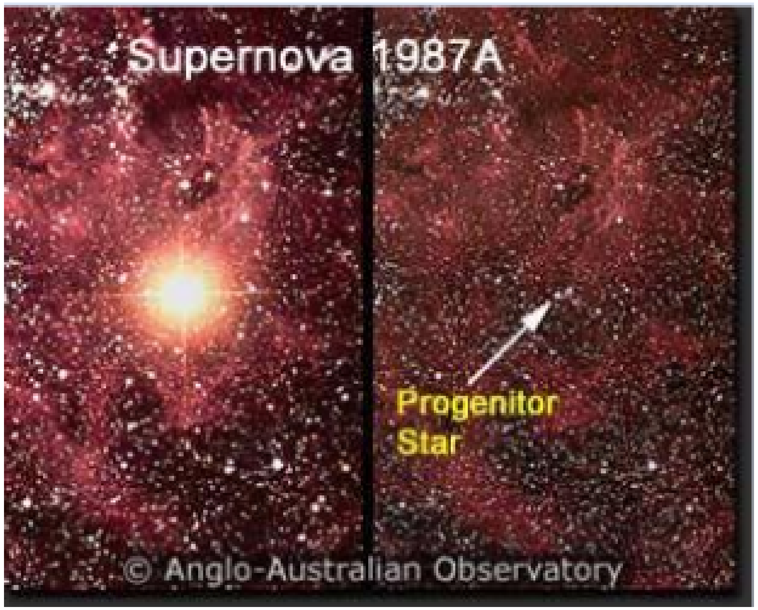
# PLAN OF TALK

- *Introduction*
- *Fast Oscillations*
- *Nonlinear Analysis*
- *Conclusion*

# CORE-COLLAPSE SUPERNOVA

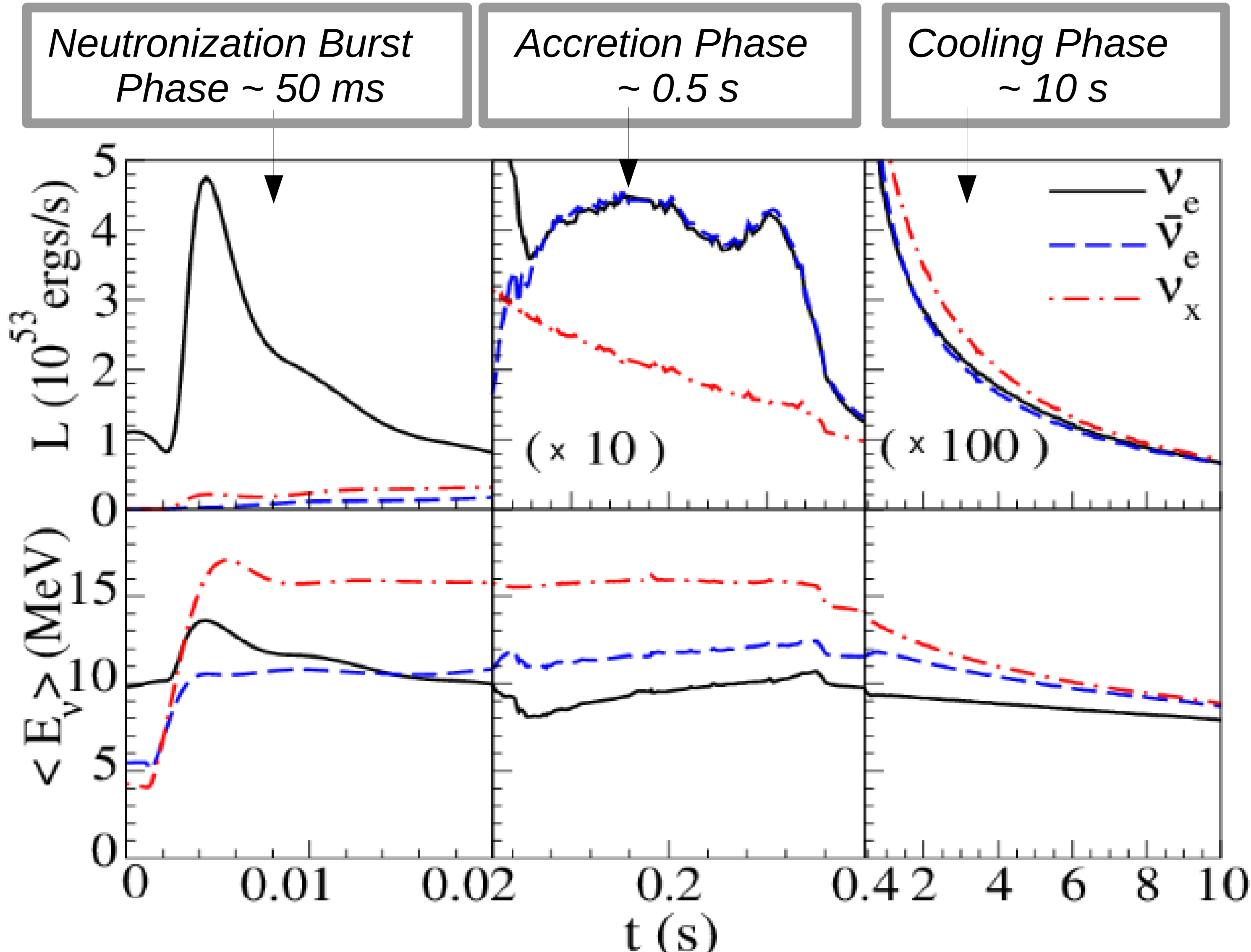



- Occurs from the death of a massive star > 8 $M_{sun}$
- Also known as the type II supernova
- 99% of Gravitational energy emitted as neutrinos($\sim 10^{58}$)
- Average energy of neutrinos around 10 MeV emitted within 10 seconds

*NEUTRINOS : RICH SOURCE OF INFORMATION*

*H. A. Bethe, Rev. Mod. Phys. 62, 801 (1990)*

# NEUTRINO EMISSION PHASES

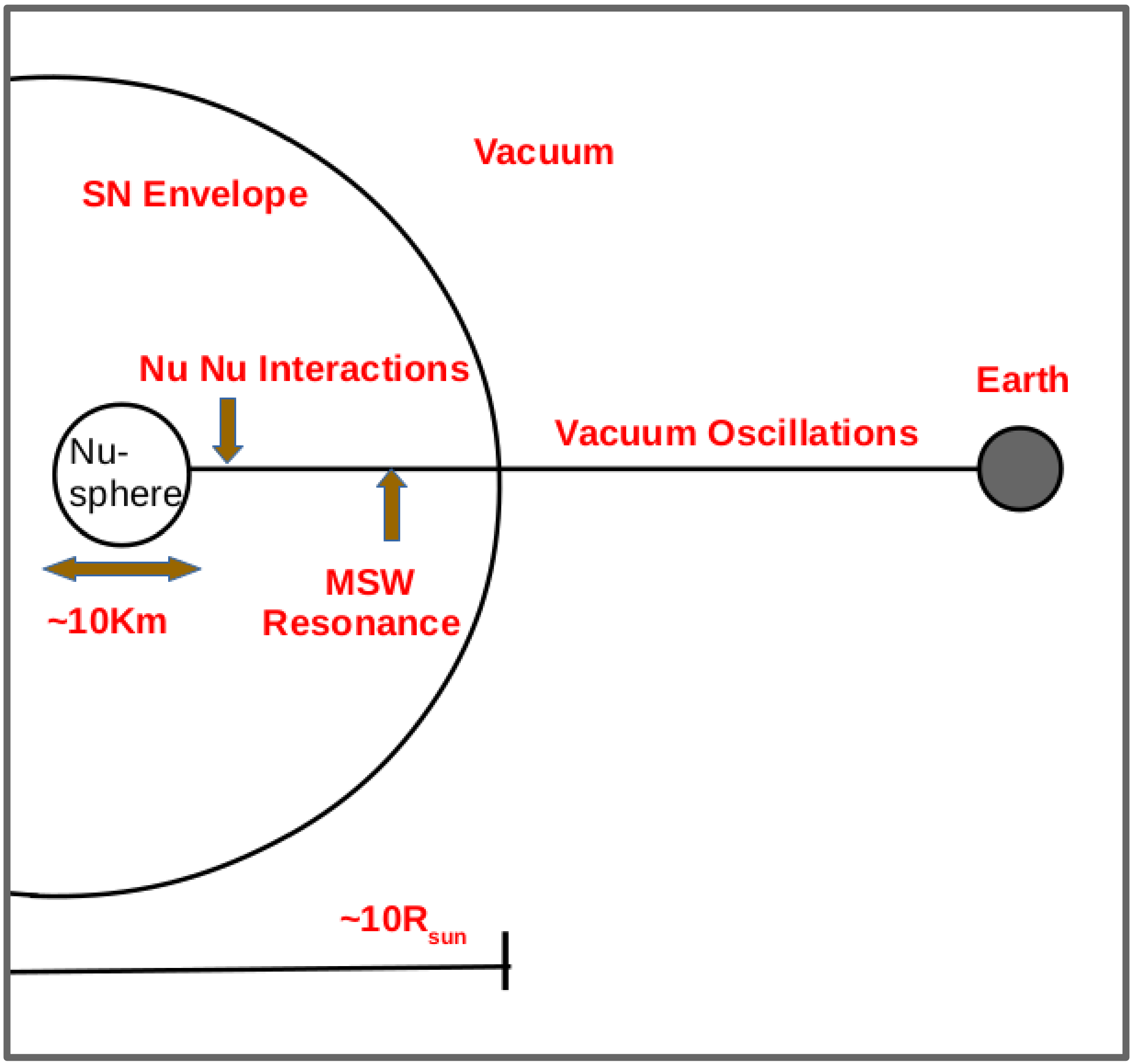



*Obtained from the results of the Basel/Darmstadt simulation*

*of a 18* $M_{sun}$ *progenitor SN*

*Chakraborty et al Phys. Rev. D95 (2017)*

# NEUTRINOS AFTER SUPERNOVA EXPLOSION



# EVOLUTION OF NEUTRINOS

$$v^{\beta}\partial_{\beta}\rho_p = -i[H_p, \rho_p]$$

$\beta$=0,1,2,3

$$H = H_{vac} + H_{matter} + H_{\nu\nu}$$

*Responsible for Collective Oscillations*

# EVOLUTION OF NEUTRINOS

$$v^{\beta}\partial_{\beta}\rho_p = -i[H_p, \rho_p]$$

$$H = H_{vac} + H_{matter} + H_{\nu\nu}$$

*Responsible for Collective Oscillations*

$$\rho_p = \begin{pmatrix} \rho_p^{ee} & \rho_p^{e\mu} & \rho_p^{e\tau} \\ \rho_p^{\mu e} & \rho_p^{\mu\mu} & \rho_p^{\mu\tau} \\ \rho_p^{\tau e} & \rho_p^{\tau\mu} & \rho_p^{\tau\tau} \end{pmatrix}$$

*Responsible for flavor conversion*

*Correspond to overall flavor content*

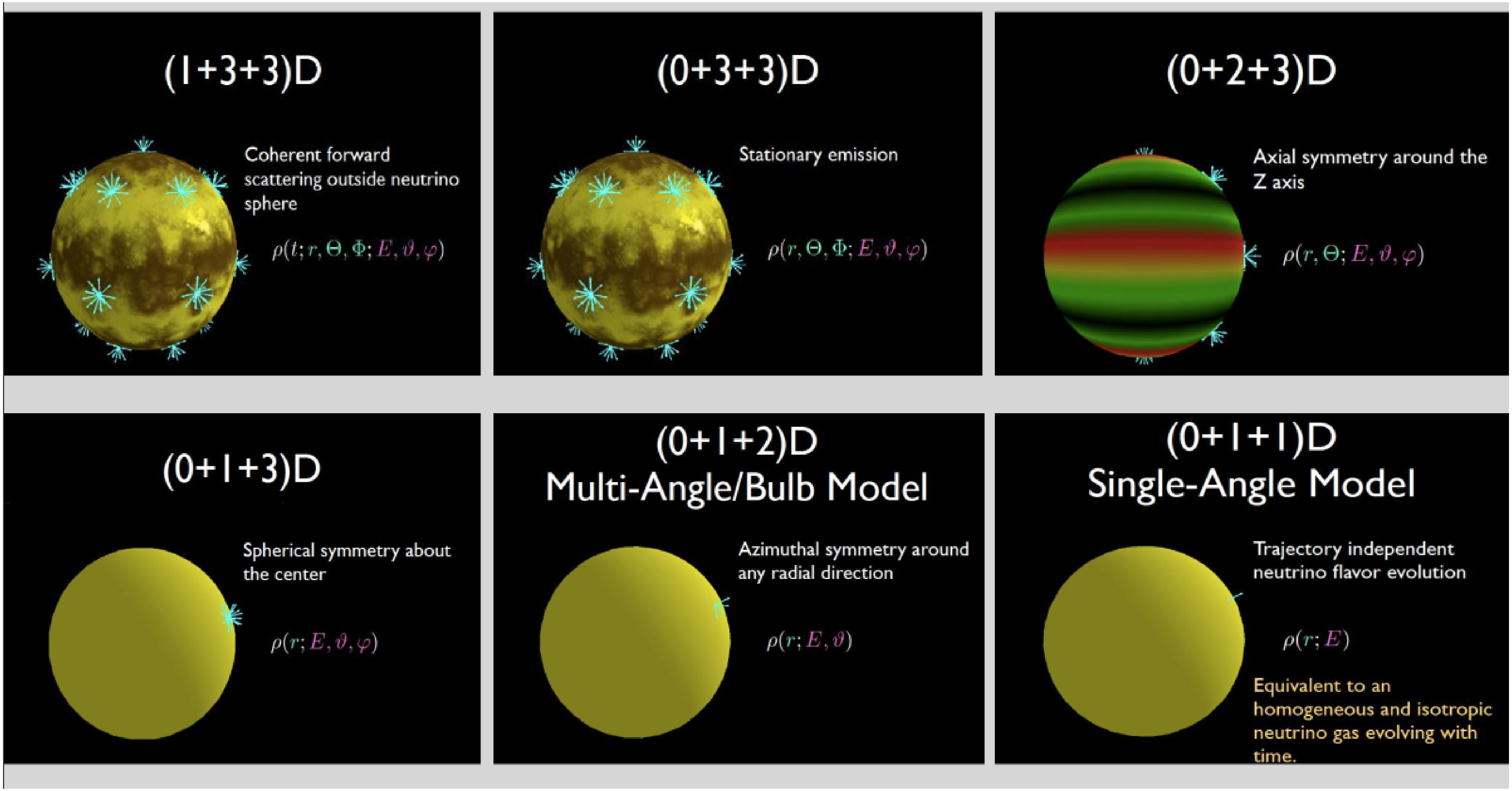
$$H_{vac} = \frac{1}{2E} diag(m_1^2, m_2^2, m_3^2)$$

$$H_{matter} \propto diag(n_e - n_{\bar{e}}, n_{\mu} - n_{\bar{\mu}}, n_{\tau} - n_{\bar{\tau}})$$

$$H_{\nu\nu} = \sqrt{2} G_F v_{\beta} \int d\,\mathbf{p}\, v^{\beta} (\rho_p - \overline{\rho}_p)$$

*Makes evolution non linear*

# 7 DIMENSIONAL PROBLEM (1+3+3)




slides from H. Duan

Duan & Shalgar, PLB 2015
Mirizzi, Mangano & Saviano, PRD 2015

# TWO FLAVOR CASE

$$i v^{\beta} \partial_{\beta} \rho_{\boldsymbol{p}}^{e\mu} = \left[ \frac{m_1^2 - m_2^2}{2E} + v_{\beta} (\lambda_e^{\beta} - \lambda_{\mu}^{\beta}) \right] \rho_{\boldsymbol{p}}^{e\mu} - \sqrt{2} G_F (\rho_{\boldsymbol{p}}^{ee} - \rho_{\boldsymbol{p}}^{\mu\mu}) v^{\beta} \int d\boldsymbol{p}' v'_{\beta} (\rho_{\boldsymbol{p}'}^{e\mu} - \bar{\rho}_{\boldsymbol{p}'}^{e\mu})$$

$$\rho_{\mathbf{p}} = \frac{f_{\nu_e,\mathbf{p}} + f_{\nu_\mu,\mathbf{p}}}{2} \mathbb{1} + \frac{f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}}}{2} \begin{pmatrix} s_{\mathbf{p}} & S_{\mathbf{p}} \\ S_{\mathbf{p}}^{*} & -s_{\mathbf{p}} \end{pmatrix}$$

$$i v^{\beta} \partial_{\beta} S_{E,v} = (\omega + v^{\beta} \lambda_{\beta}) S_{E,v} - \sqrt{2} G_F v^{\beta} \int d\Gamma' v'_{\beta} ( f_{\nu_e, v'} - f_{\bar{\nu}_e, v'}) S_{E',v'}$$

$$\int d\Gamma' = \int_{-\infty}^{\infty} dE' \int_0^1 du' \int_0^{2\pi} d\phi'$$

*S. Airen et al, JCAP 1812 (2018)*

# *SLOW OSCILLATIONS*

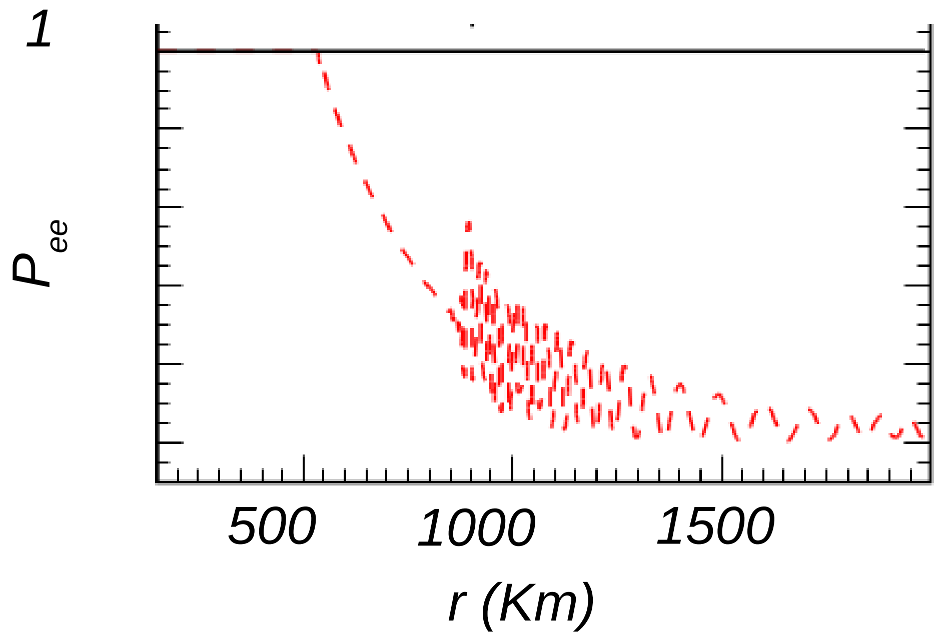
- *Driving frequency - ω*

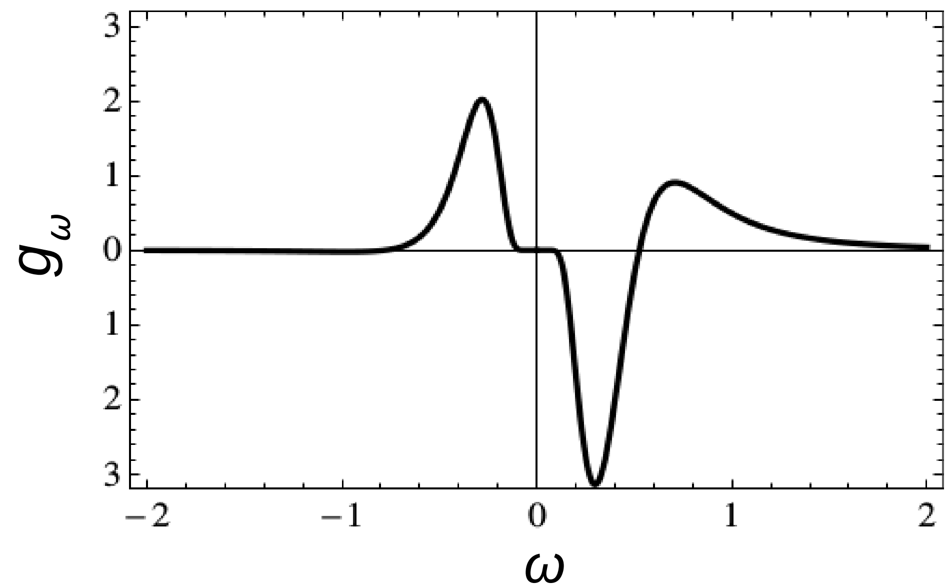
$$\omega = \frac{\Delta m^2}{2E} \sim O\left(1\, km^{-1}\right)$$

- *Occurs ~ 10² km from the neutrinosphere*

- *Growth Rate ~ ω*

- *Spectrum only energy dependent*



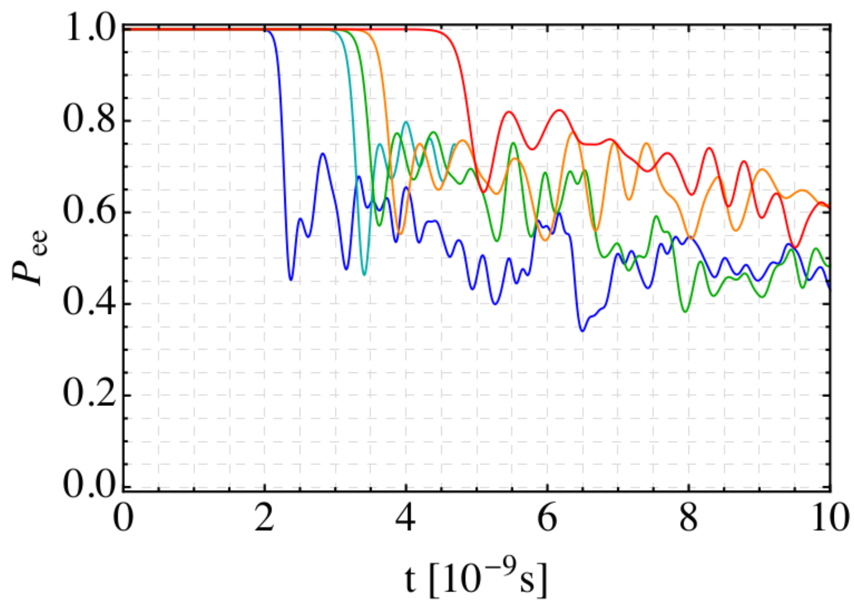

*Chakraborty et al, arXiv:1105.1130 ; A. Banerjee et al, arXiv:1107.2308*

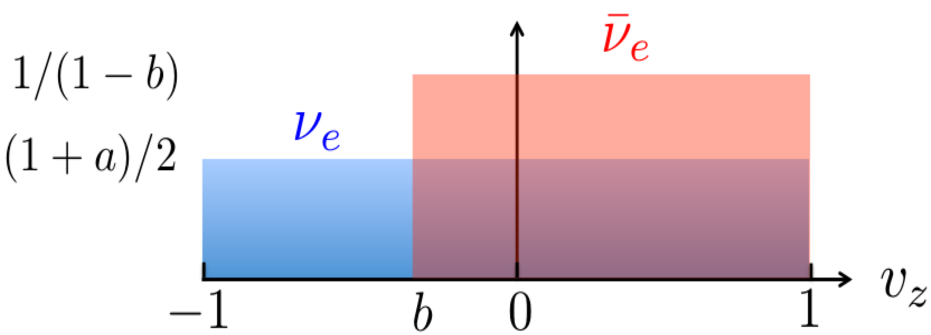
# FAST OSCILLATIONS

- *Driven by neutrino density μ*

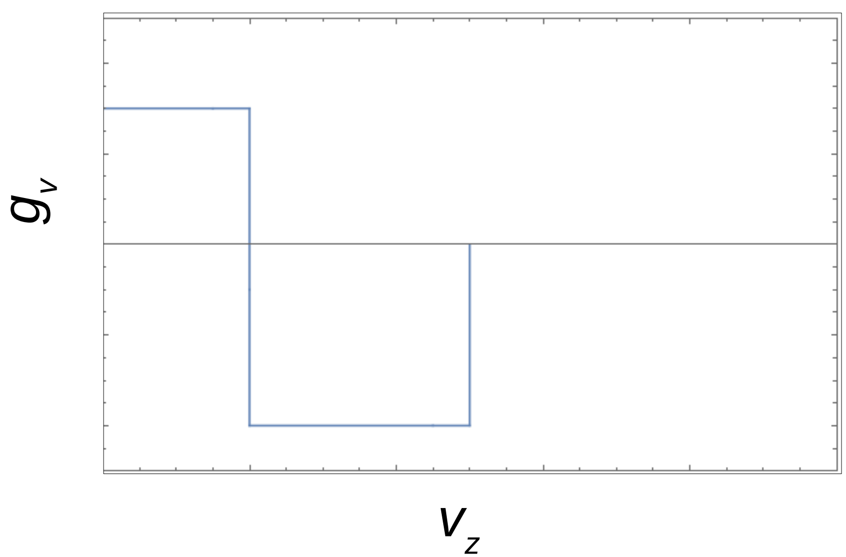
$$\mu \sim \sqrt{2} G_F n_\nu \sim O(10^5 km^{-1})$$

- *May occur just above the neutrinosphere*
- *Growth Rate ~ μ*
- *Spectrum dependent on angular distribution*




*B. Dasgupta et al arXiv:1609.00528*




*R. F. Sawyer Phys. Rev. D 72,045003 (2005)*

*Chakraborty et al, arXiv:1602.00698*

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# FAST OSCILLATIONS

$$i v^{\beta} \partial_{\beta} S_{E,v} = (\omega + v^{\beta} \lambda_{\beta}) S_{E,v} - \sqrt{2} G_F v^{\beta} \int d\Gamma' v'_{\beta} (f_{\nu_e, v'} - f_{\bar{\nu}_e, v'}) S_{E',v'}$$

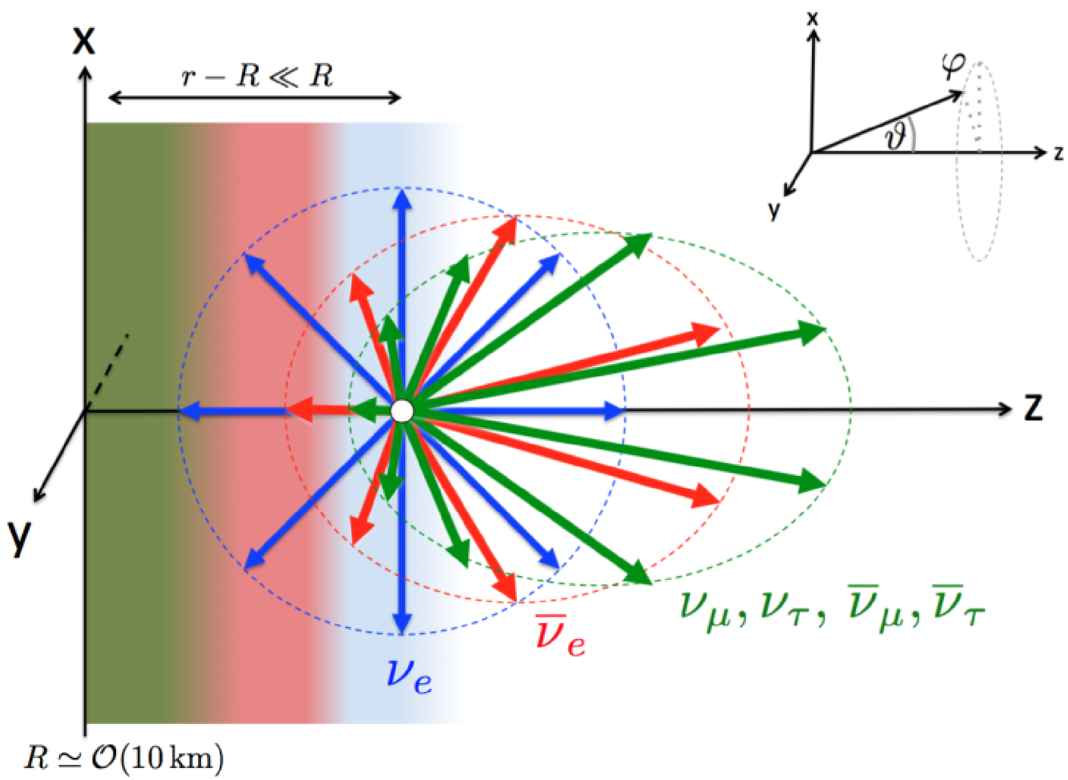
- $M^2 = 0$ i.e. $\omega_j \rightarrow 0$
- Growth Rate ~ μ
- Spectrum dependent on angular distribution
- Crossing in the spectrum important

$$i v^{\beta} \partial_{\beta} S_{v} = (v^{\beta} \lambda_{\beta}) S_{v} - v^{\beta} \int d\mathbf{v}' v'_{\beta} G_{\mathbf{v}} S_{v'}$$

*I. Izaguirre et al, Phys. Rev. Lett. 118, 021101(2017)*

## Fast ocillations study – Angular distributions

## Different Flavors – Different angular distributions




*Flavor dependent zenith angle distributions of neutrino fluxes*

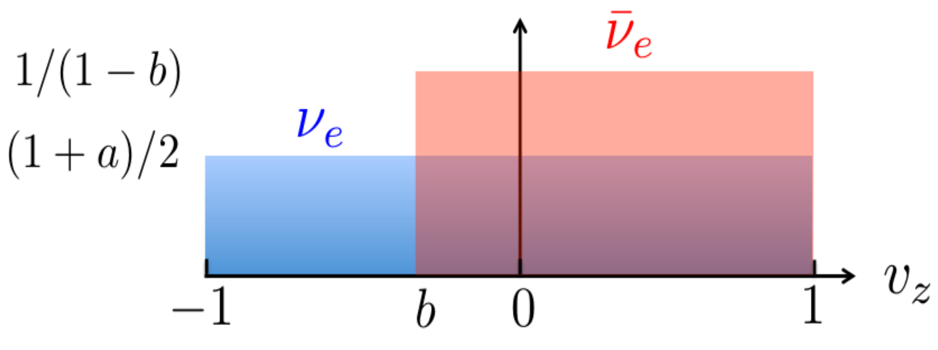
*B. Dasgupta et al arXiv:1609.00528*

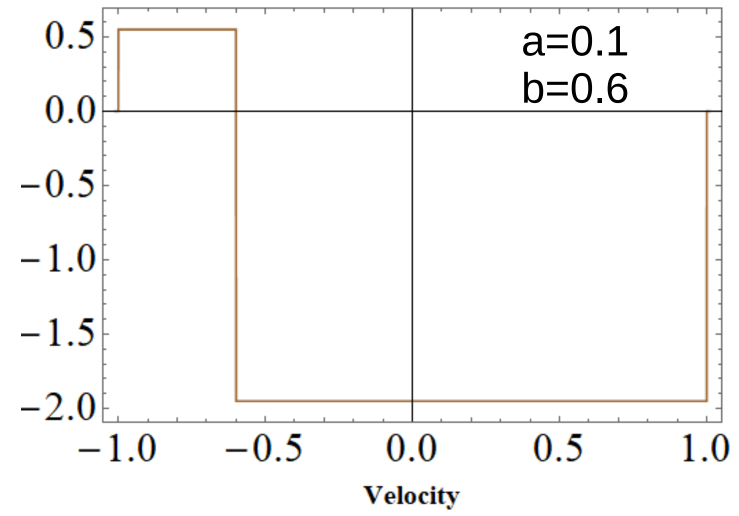
# FAST OSCILLATIONS : TWO FLAVOR

*ELN is given by:*

$$G_{\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left( f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} \right)$$

*Zero Crossing in $G_v$* ⟶ *Necessary condition for fast flavor conversions*



# THREE FLAVOR CASE

$$i v^{\beta}\partial_{\beta}\rho_{\boldsymbol{p}}^{e\mu}=\left[\frac{m_1^2-m_2^2}{2E}+v_{\beta}(\lambda_e^{\beta}-\lambda_{\mu}^{\beta})\right]\rho_{\boldsymbol{p}}^{e\mu}-\sqrt{2}G_F(\rho_{\boldsymbol{p}}^{ee}-\rho_{\boldsymbol{p}}^{\mu\mu})v^{\beta}\int d\boldsymbol{p}'\,v'_{\beta}(\rho_{\boldsymbol{p}'}^{e\mu}-\bar{\rho}_{\boldsymbol{p}'}^{e\mu})$$

*Similar equations for e-τ and μ-τ*

$$\rho_{\mathbf{p}}=\frac{f_{\nu_e,\mathbf{p}}+f_{\nu_\mu,\mathbf{p}}+f_{\nu_\tau,\mathbf{p}}}{3}\mathbb{1}+\frac{f_{\nu_e,\mathbf{p}}-f_{\nu_\mu,\mathbf{p}}}{3}\begin{pmatrix} s_{\mathbf{p}} & S_{1\mathbf{p}} & 0\\ S_{1\mathbf{p}}^* & -s_{\mathbf{p}} & 0\\ 0 & 0 & 0\end{pmatrix}$$
$$+\frac{f_{\nu_e,\mathbf{p}}-f_{\nu_\tau,\mathbf{p}}}{3}\begin{pmatrix} s_{\mathbf{p}} & 0 & S_{2\mathbf{p}}\\ 0 & 0 & 0\\ S_{2\mathbf{p}}^* & 0 & -s_{\mathbf{p}}\end{pmatrix}+\frac{f_{\nu_\mu,\mathbf{p}}-f_{\nu_\tau,\mathbf{p}}}{3}\begin{pmatrix} 0 & 0 & 0\\ 0 & s_{\mathbf{p}} & S_{3\mathbf{p}}\\ 0 & S_{3\mathbf{p}}^* & -s_{\mathbf{p}}\end{pmatrix}$$

$$i v^{\beta}\partial_{\beta}S_{jE,v}=(\omega_j+v^{\beta}\lambda_{j\beta})S_{jE,v}-\sqrt{2}G_F v^{\beta}\int d\Gamma_j'\,v'_{\beta}\,g_{jE',v'}S_{jE',v'}$$ j=1,2,3

$$\int d\Gamma'=\int_{-\infty}^{\infty}dE'\int_0^1 du'\int_0^{2\pi}d\phi'$$

*M. Chakraborty, S. Chakraborty, JCAP01(2020)005*

# ANGULAR SPECTRUM

*In Fast flavor limit,*

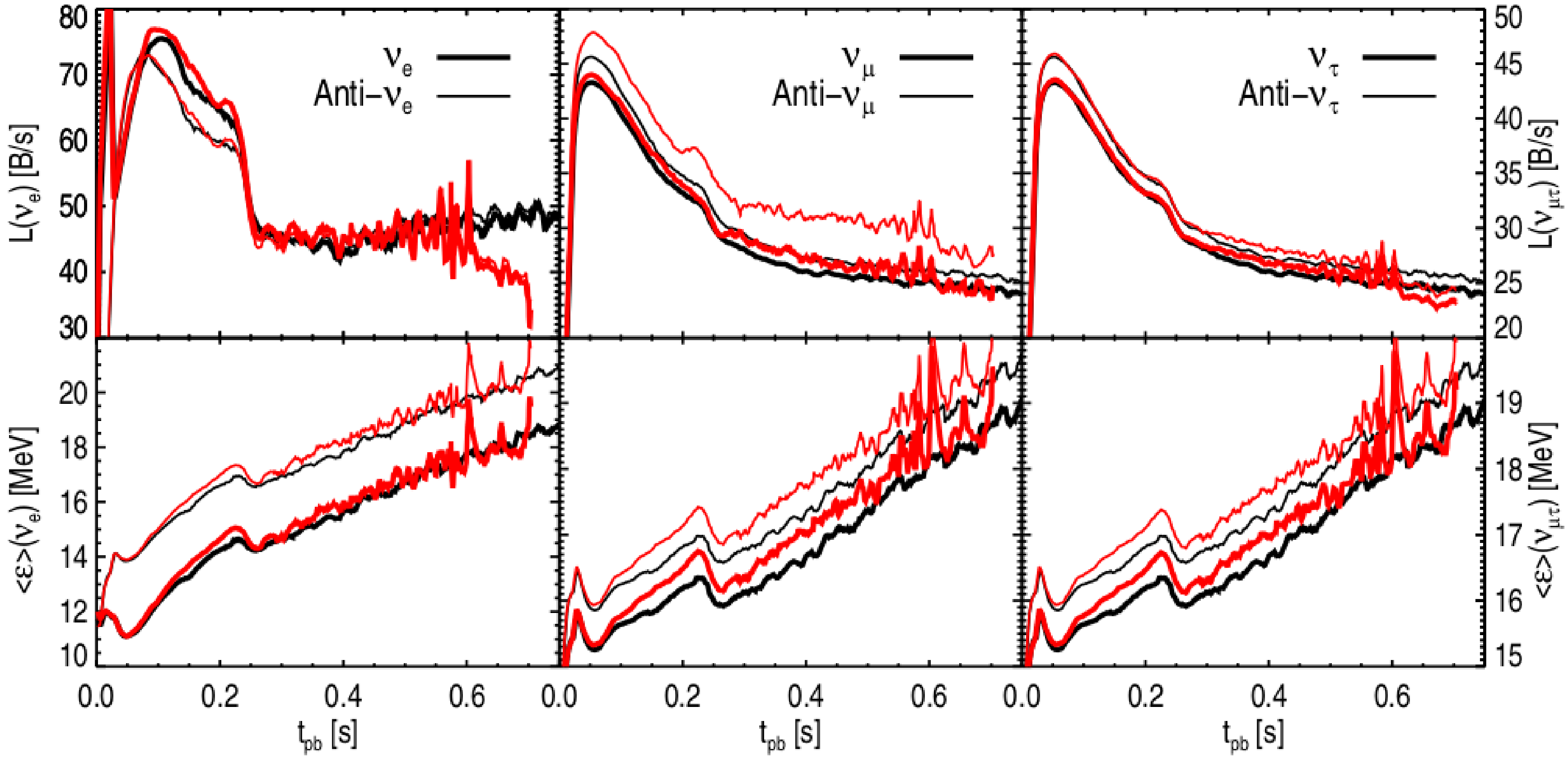
$$G_{1,\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2}\, g_{1,E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + f_{\bar{\nu}_\mu,\mathbf{p}}\right)$$

$$G_{2,\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2}\, g_{2,E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}}\right)$$

$$G_{3,\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2}\, g_{3,E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_\mu,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}}\right)$$

- *MuLN and TauLN in addition to ELN*
- *In Case of two flavor,* $f_{\nu_\mu} = f_{\bar{\nu}_\mu},\ f_{\nu_\tau} = f_{\bar{\nu}_\tau}$
- *The spectrum reduces to only ELN*

# EFFECT OF PRESENCE OF MUONS




*Obtained from 2D simulations(R. Bollig et al, arXiv:1706.04630)*

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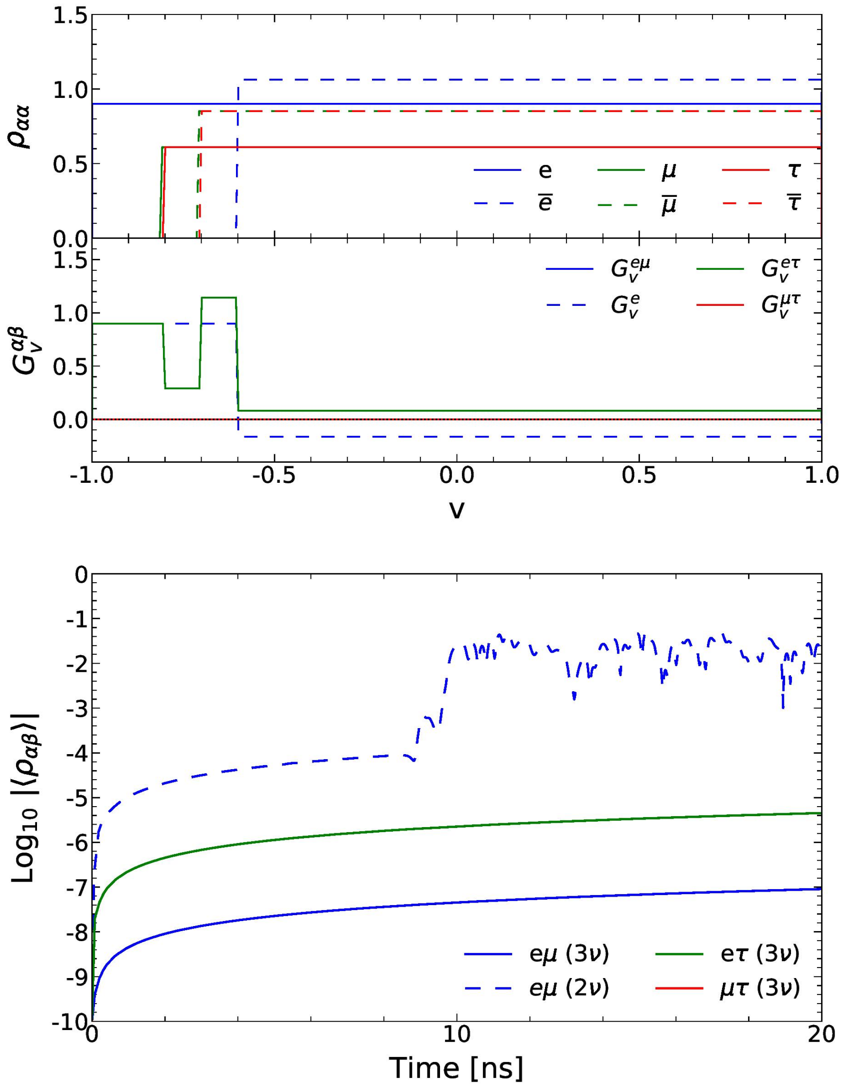
# FAST OSCILLATIONS : THREE FLAVOR

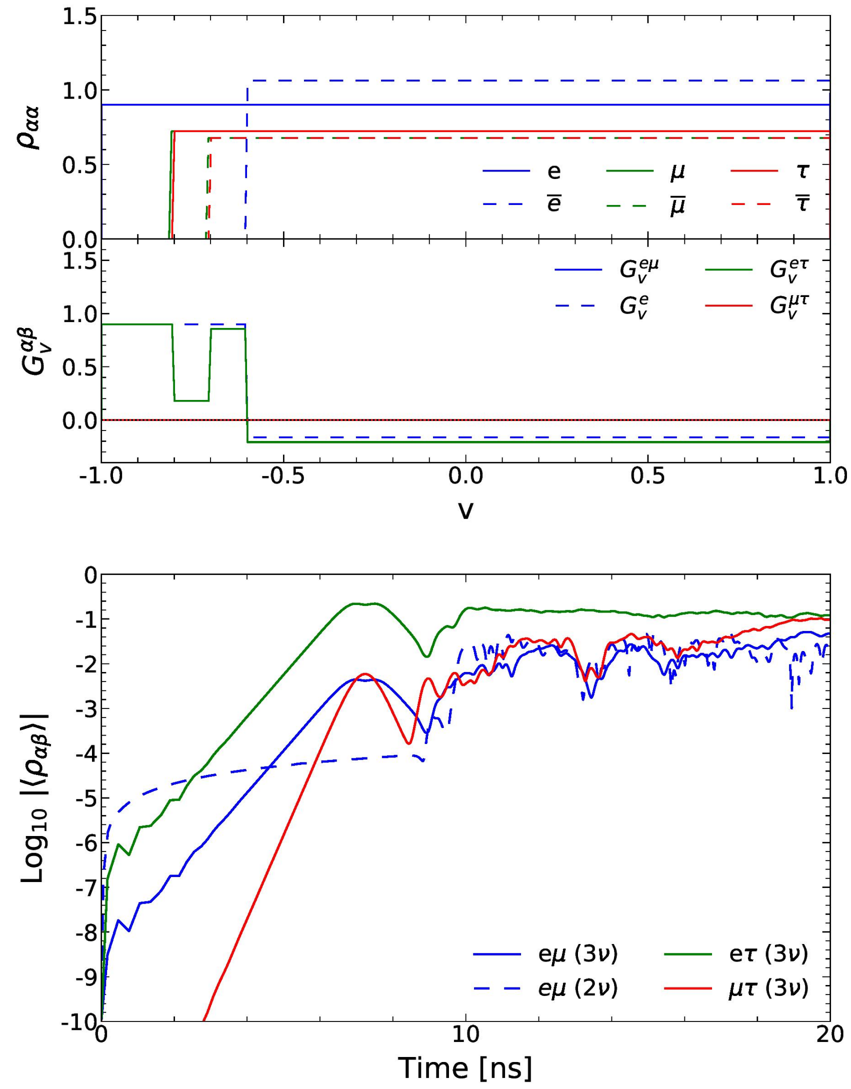
*Based on PRL 125,251801 (2020)*
*Authors : F.Capozzi, **M.C**, S.Chakraborty and M.Sen*

- *Muons may be present in early accretion phase (R. Bollig et al, arXiv:1706.04630)*
- *Increases the luminosities and average energies of other two flavors*
- *Study of fast oscillations in non linear picture*
- *Effect of addition of third flavor*

# NON-LINEAR ANALYSIS : NUMERICAL EXAMPLES

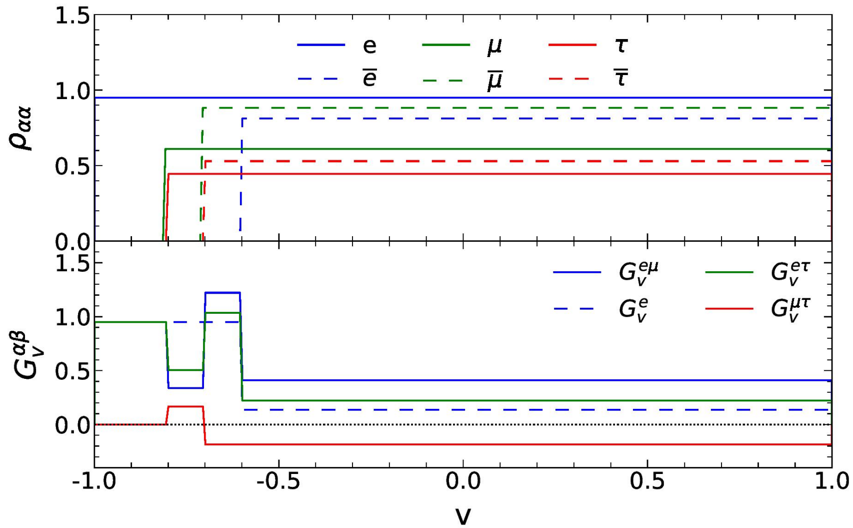


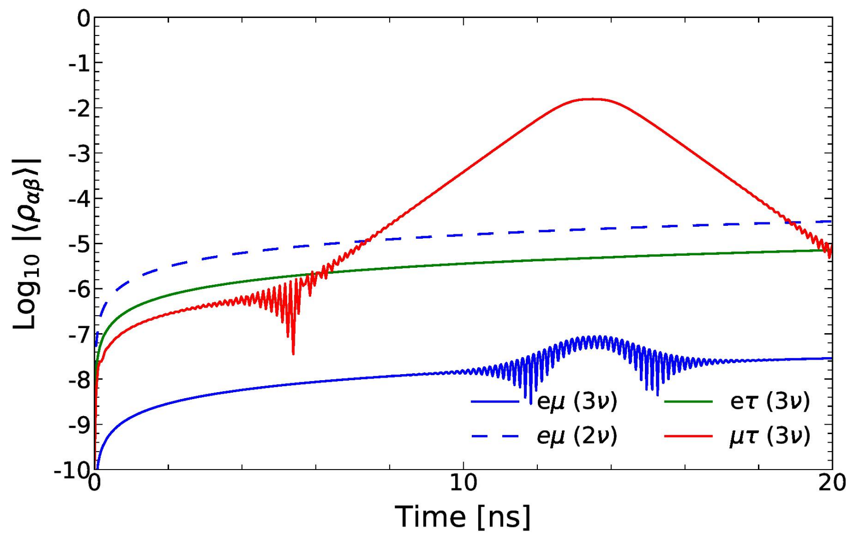

*Case 1*

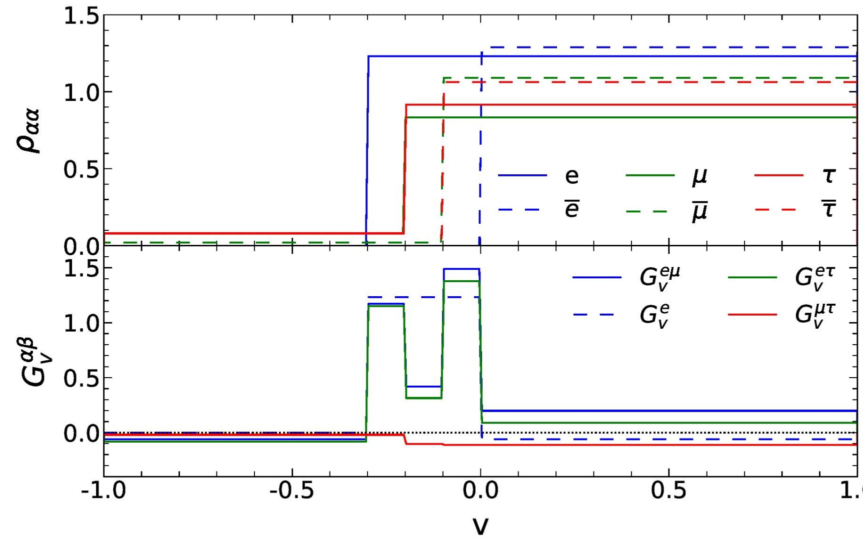


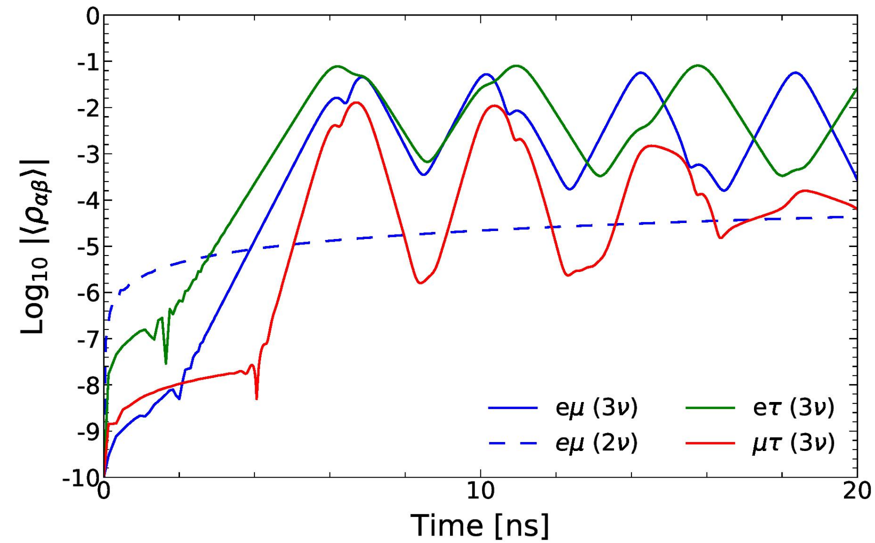

*Case 2*

# NON-LINEAR ANALYSIS : NUMERICAL EXAMPLES




*Case 3*

*Case 4*

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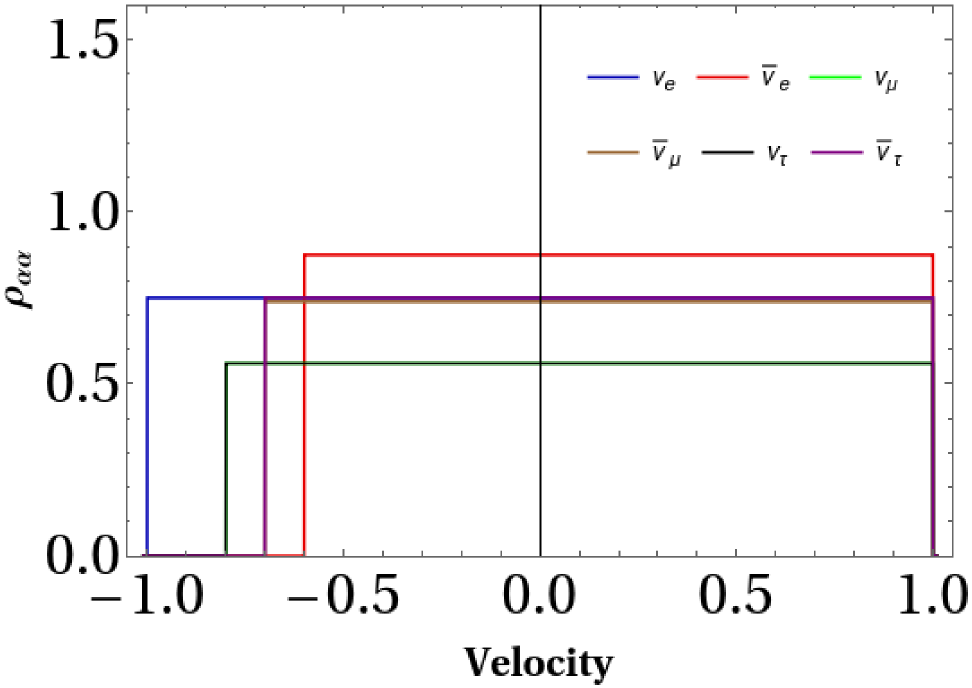
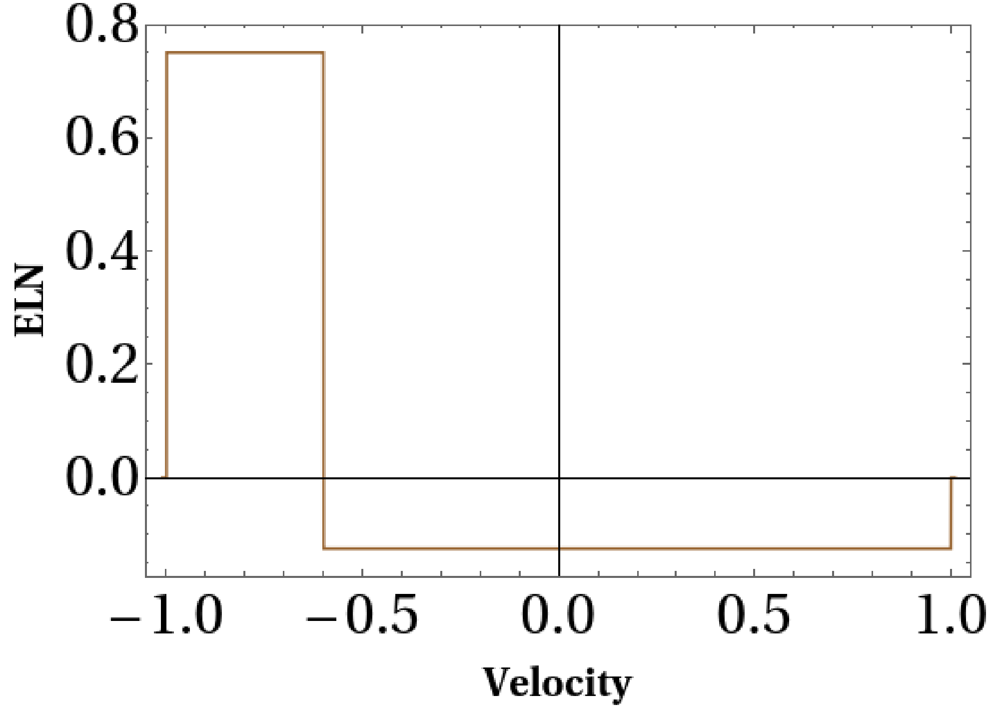
# CONCLUSION

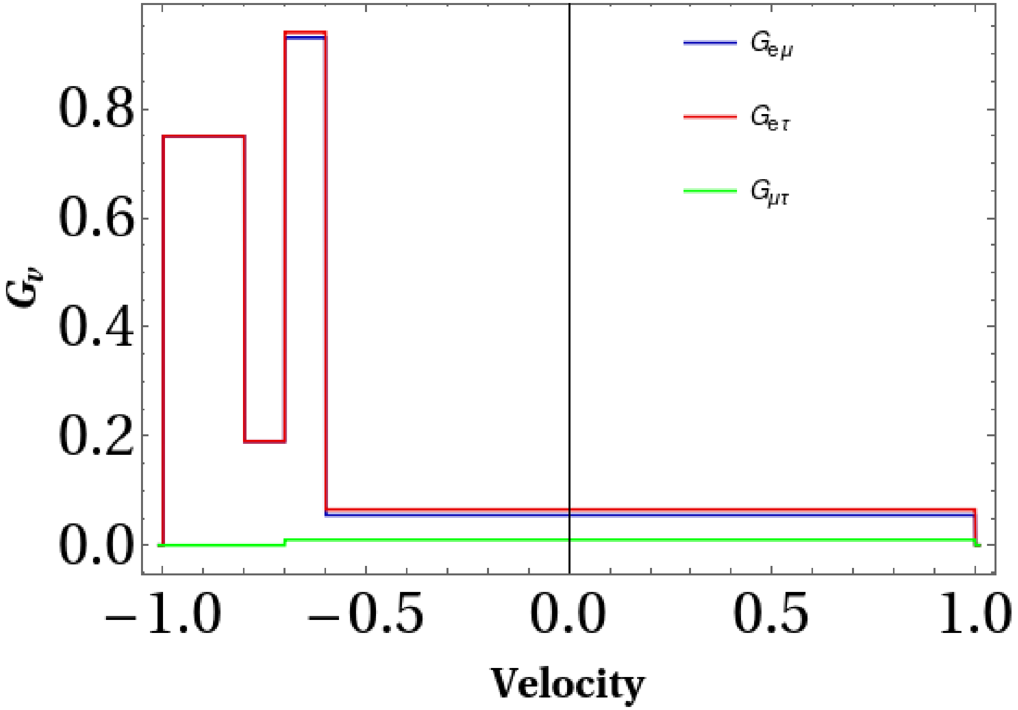
- *Non-negligible MuLN and TauLN can enhance or supress the crossings in ELN*
- *Effect on Fast flavor conversions*
- *Nonlinear analysis done*
- *Simple toy examples considered*
- *Explain the importance of need of detailed 3 flavor analysis*
- *Future muon simulations will further explain the effect in case of realistic situations*

# THANK YOU

# CASE 1

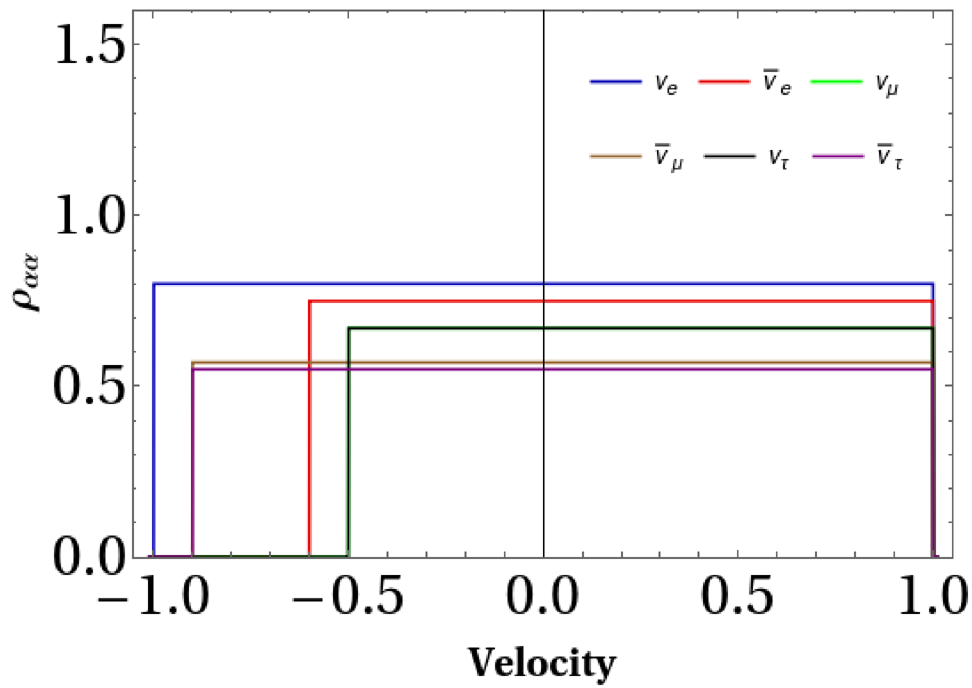


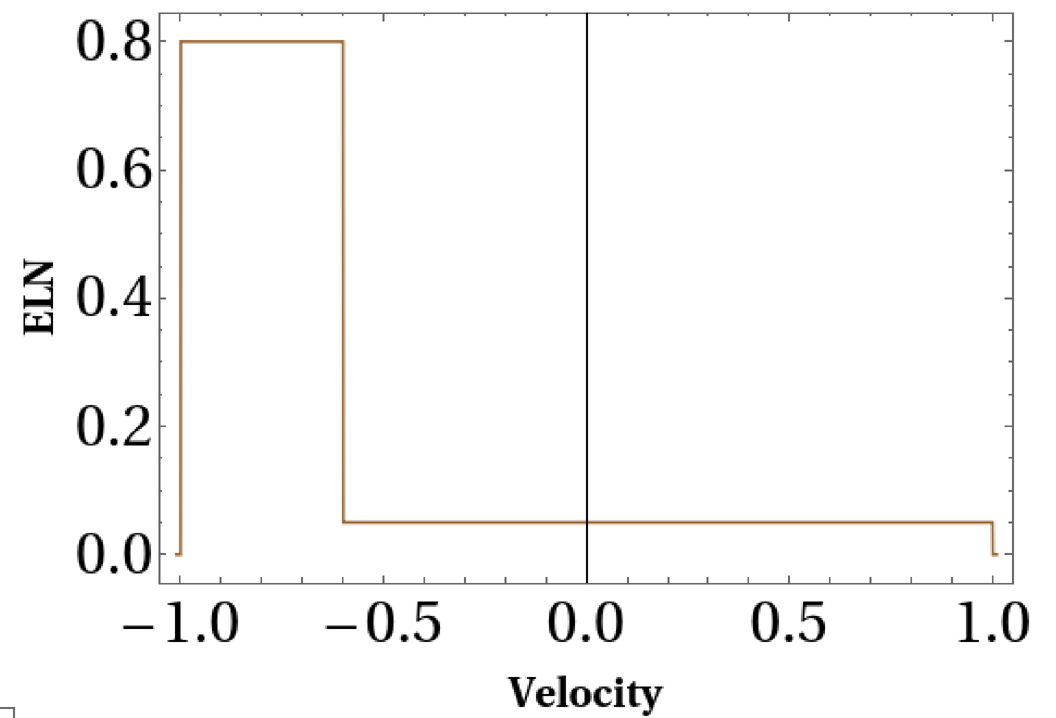



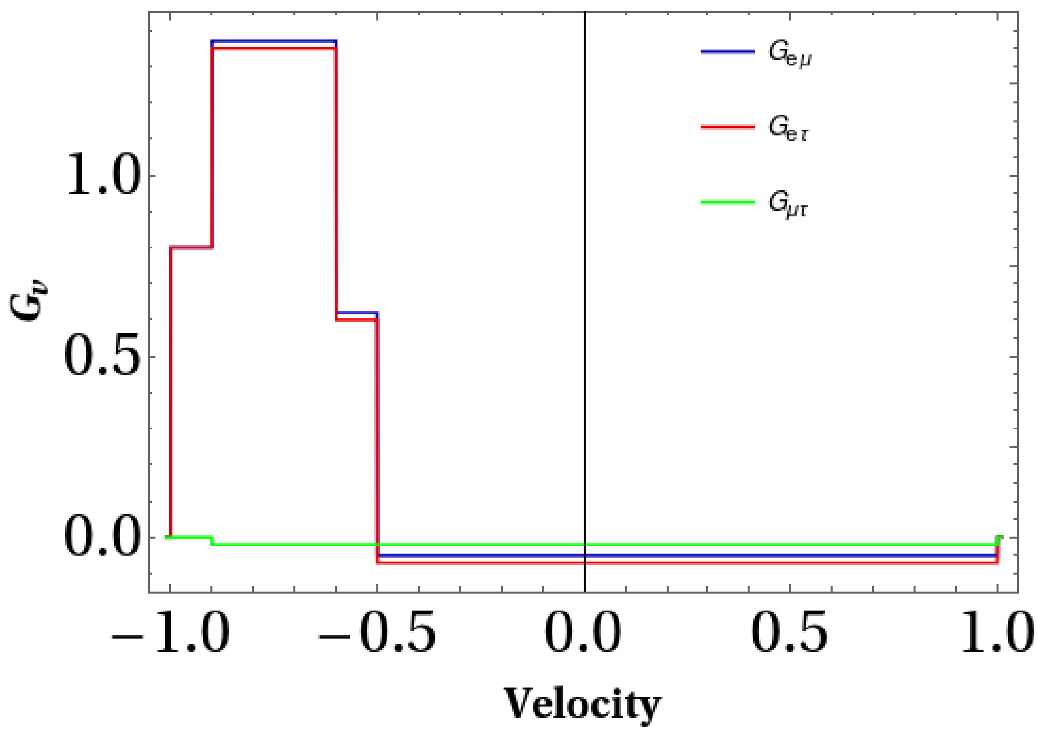




No instability in any sector from linear stability analysis

# CASE 2

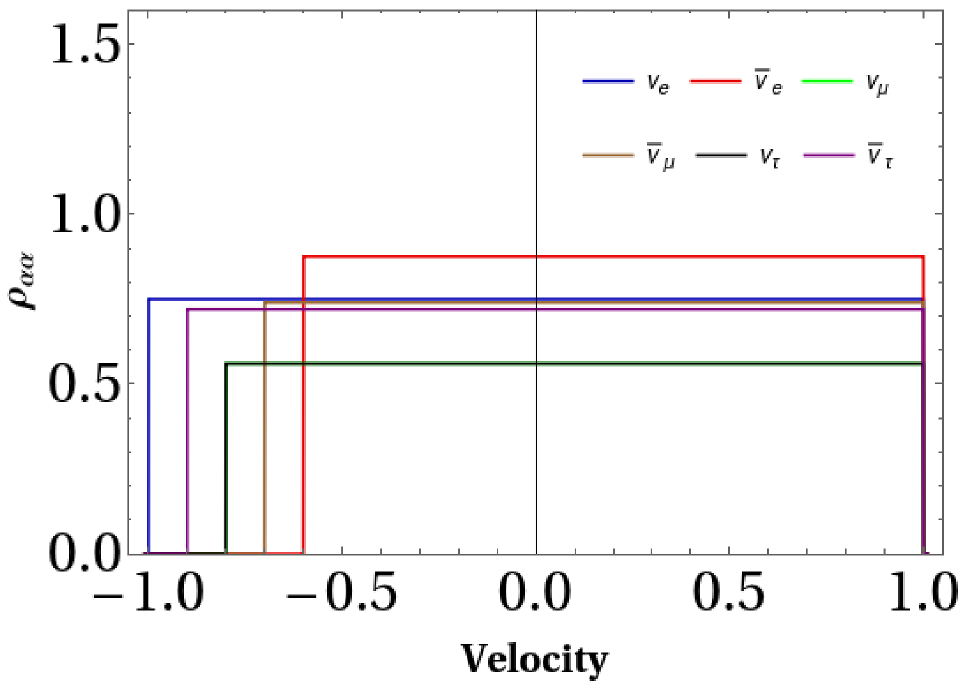


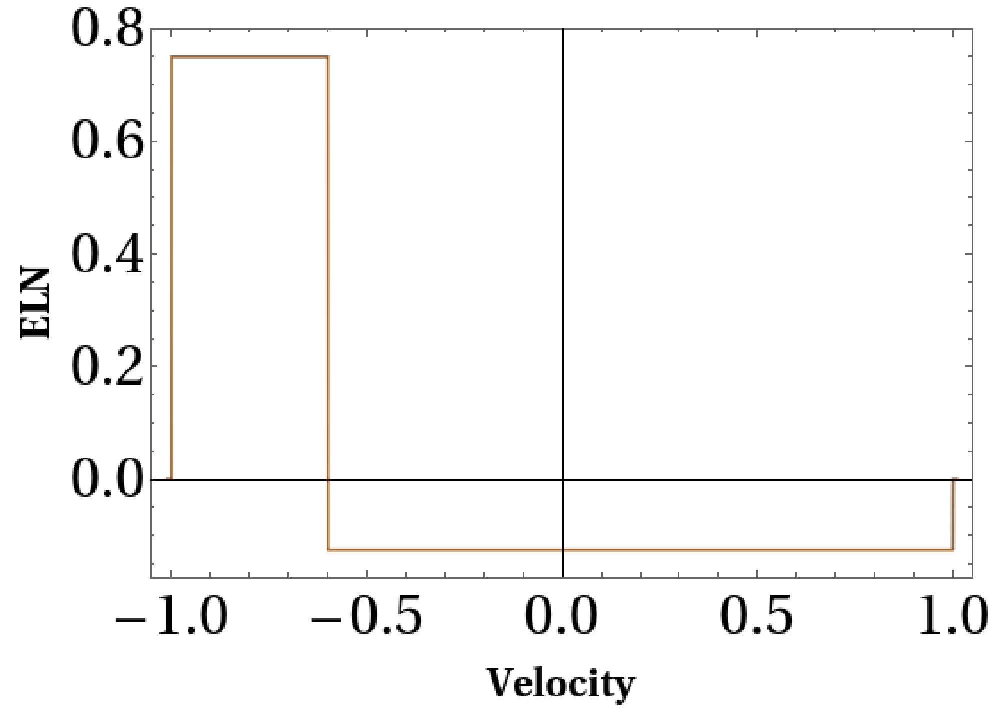



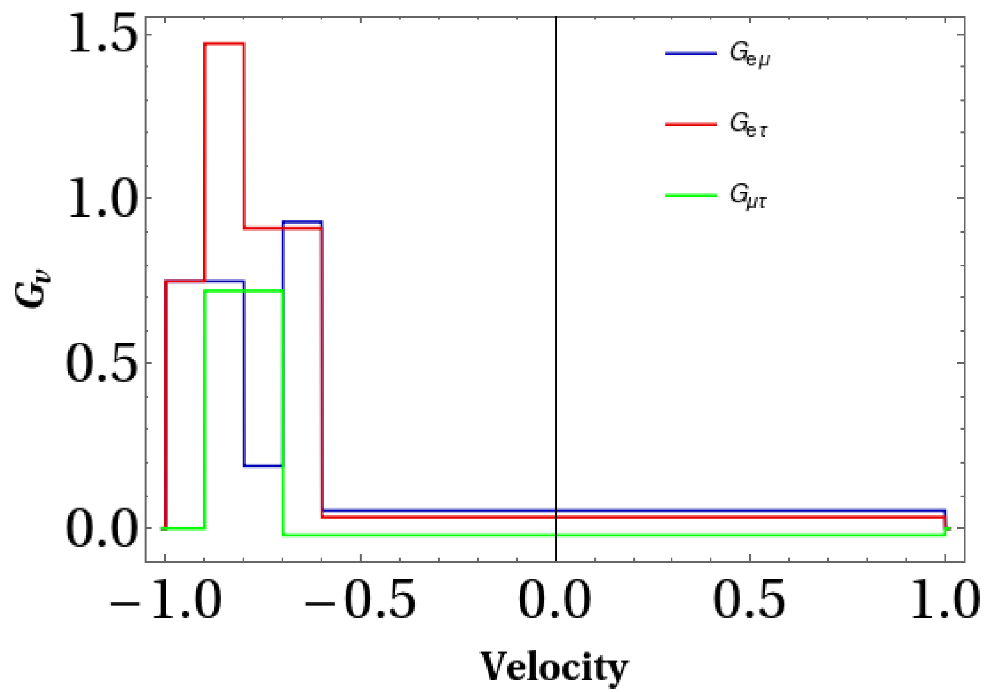




| Sector | κ/μ |
|---|---|
| e-μ | 0.017 |
| e-τ | 0.025 |
| μ-τ | No Instability |

# CASE 3




| Sector | κ/μ |
| --- | --- |
| e-μ | No Instability |
| e-τ | No Instability |
| μ-τ | 0.006 |

# NON-LINEAR ANALYSIS : 3 FLAVOR

$$i\left(\partial_t + \mathbf{v}_\mathbf{p} \cdot \nabla_\mathbf{x}\right) \rho_{\mathbf{p},\mathbf{x},t} = \left[\Omega_{\mathbf{p},\mathbf{x},t}, \rho_{\mathbf{p},\mathbf{x},t}\right]$$

*Vacc.+matter+ νν interaction*

- *Fast Oscillation - Vaccum term acts as seed*
- *Only temporal evolution considered*
- *Matter terms neglected*
- *Dynamics of the system governed by nonlinear terms (νν interaction)*